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Fiber Optic Electrometer with Mechanical Resonator

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Abstract

A fiber optic electrometer with oscillating element is described in this paper. The non-contact interferometric interrogation system of electrometer provides high-sensitivity measurement of voltage and charge, inducing very small disturbances in the measured object. The sensitivity of $2 \cdot 10^{-4} V/(Hz)^{1/2}$ was achieved, while the sensitivity of $3\ \mu V/(Hz)^{1/2}$ is predicted from the thermal fluctuation limit. An order of magnitude estimations indicate that the sensitivity to an electric charge close to the electron charqe may be achieved with electrometer of considered type.

1. Introduction

Up to present time the interferometric optical systems have found a wide application in research and industry as providing the sensitivity to displacements as small as parts of angstrom. New possibilities for development of resonator-based devices have appeared with creation of fiber optic interferometers[1-3]. In this paper a new fiber optic electrometer with oscillating element is considered. The main advantages of this device are high electrical resistance ($> 10^{12}\ \Omega$) and small capacity (< 1 pF) that allows to measure the voltage and the charge while inducing extremely small disturbances in the measured object.

In many cases the physical limitation for the sensitivity of interferometer-based sensors is related to the presence of thermally generated vibrations[4,5]. It is shown in this paper that the thermal vibrations of the sensitive resonator lead to fluctuations of the electrometer output signal comparable to the noises of other types. The expressions for the electrometer transconductance and threshold sensitivity are derived. The ways for improving of the technical characteristics are discussed.

2. Principle of operation

The simplest scheme of electrometer was realized in this work (Fig.1). The resonator in the form of cantilever beam made of metallic glass is placed in the center of the gap formed by the capacitor plates. The alternating voltage is applied to the capacitor so that the alternating electrical field emerges between the plates. When the measured voltage u_0 is applied to the resonator, the electrical charge appears on it. In this case the cantilever beam is acted upon by the alternating electrostatic force, giving rise to excitation of resonator flexural vibrations. The amplitude of oscillation is proportional to the value of u_0.

The partially reflecting surface of the beam and the output end of the fiber form a Fabry-Perot cavity. Oscillation of the beam modulates reflectivity of this cavity so that an intensity-modulated light is propagating back to the photodetector when the cavity is illuminated with a CW laser source. The alternating electrical signal on the output of photodetector is rectified by lock-in amplifier, DC output of which will be proportional to the amplitude of cantilever beam vibrations. To improve the signal-to-noise ratio the oscillation was excited with the resonator natural frequency. In the simplest scheme of the device the mean separation between the reflectors of interferometer was adjusted applying the magnetic field perpendicular to the resonator plate.

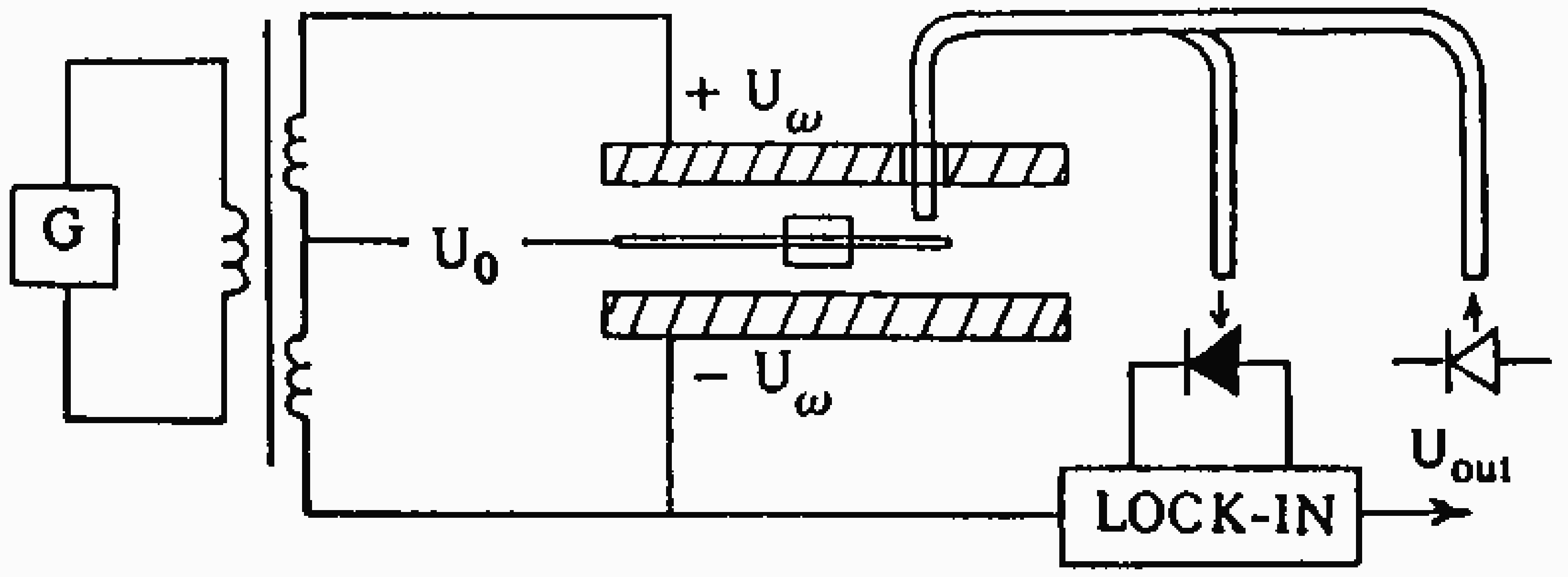

Fig.1. The scheme of fiber optic electrometer.

3. Experimental results

The mechanical resonator with length $l = 6.65$ mm, width $b = 2.5$ mm, and thickness $h = 40\,\mu$m was made of metallic glass. The met-glass was chosen as a material of the resonator because of its excellent mechanical properties, high reflectivity, and possibility to change the mean separation between interferometer mirrors by applying the magnetic field to the capacitor. The oscillation was excited with the resonator natural frequency $f_r = 672$ Hz. The oscillator quality factor $Q = 134$ was observed. The alternating voltage of amplitude $u_\omega = 56$ V was applied to the capacitor. The distance between the capacitor plates was $2d = 2.46$ mm.

Oscillation of the resonator $y(t)$ modulated the reflectivity of the Fabry-Perot cavity formed by the partially reflecting surface of the resonator and the end of single-mode fiber. Therefore, an intensity-modulated light $I(t) \sim \sin\varphi(t)$ was returned along the fiber to the photodetector when this cavity was illuminated with CW radiation of 1mW laser source with wavelength $\lambda = 0.85\,\mu$m. The resonator oscillates harmonically and, consequently, $\varphi(t) = \varphi_0 + \varphi_\omega \sin\omega t$. The value of φ_0 was adjusted by applying the magnetic field perpendicular to the resonator plate to maintain $\varphi_0 = 2\pi n$ (where n is an integer constant). The value of φ_ω is proportional to the applied voltage u_0. The coefficient of the conversion of applied voltage into the interferometer phase modulation was measured to be $S_e = \varphi_\omega / u_0 = 0.519$ rad/V.

Fig.2 shows the electrometer output signal when the measured voltage u_0 is increased temporarily by a small value of 2 mV. We can see from the figure that the output signal fluctuates even without changes in u_0. Fig.2 allows to evaluate the minimal detectable voltage u_{min} as $2 \cdot 10^{-4} \mathrm{V}/(\mathrm{Hz})^{1/2}$.

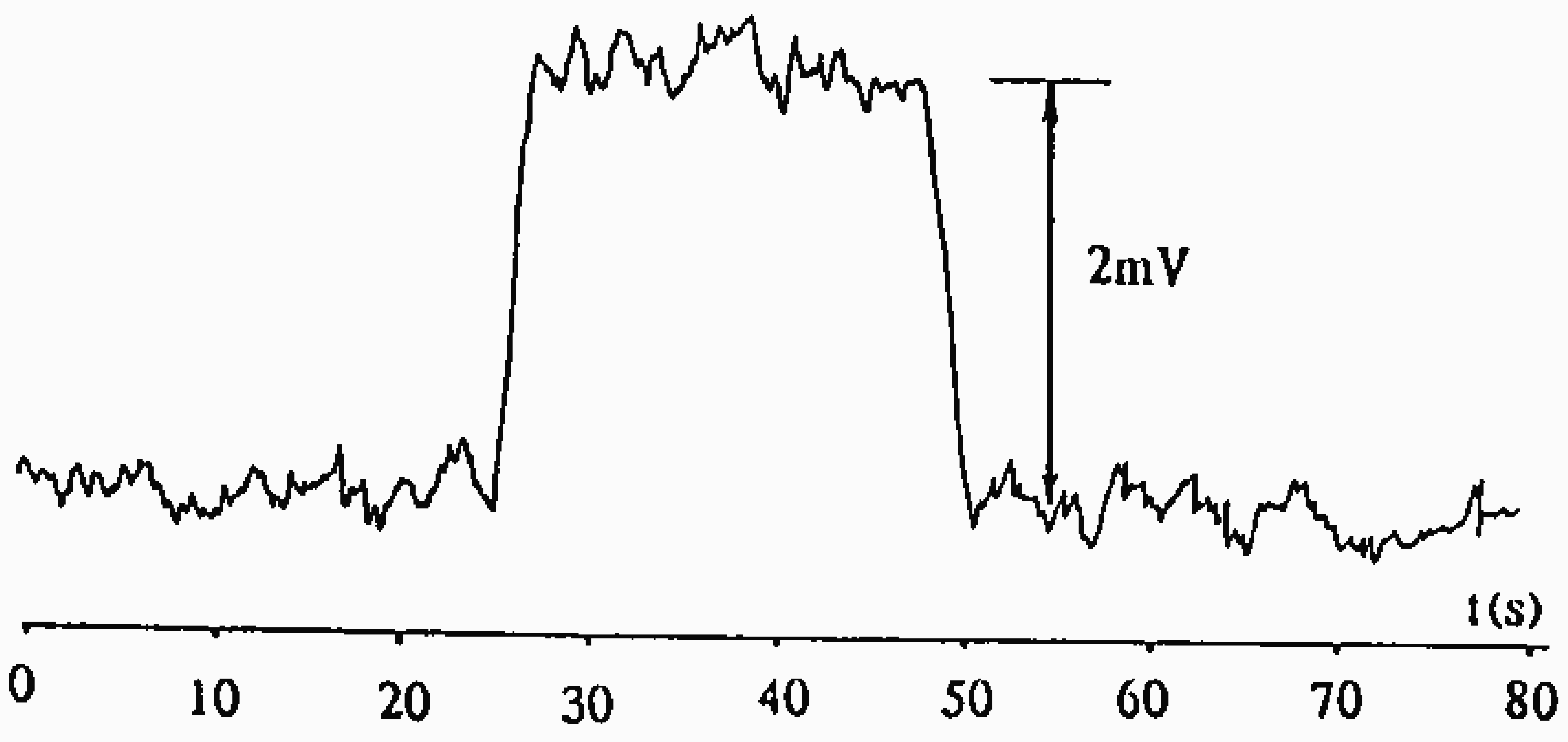

Fig.2. The response of the electrometer to a temporal change of u_0.

4. Theoretical modeling

When an alternating voltage u_ω is applied to the capacitor, the electric field E_ω appears between the capacitor plates

$$E_\omega = \frac{u_\omega}{d} \cos \omega t \tag{1}$$

where d is the distance between the resonator and one of the plates of the capacitor. Under the action of electric field E_ω the charged resonator is acted upon by a force F_ω distributed uniformly along the length of the resonator

$$F_\omega = \frac{2 \varepsilon_0 l b}{d^2} u_0 u_\omega \cos \omega t \tag{2}$$

where ε_0 is the dielectric constant.

The Fourier projection Φ of exciting force F_ω on the fundamental flexural mode of oscillation $\psi(x)$ is given by the following expression[6]

$$\Phi = \frac{\int_0^l F \cdot \psi(x)\, dx}{\int_0^l \psi^2(x)\, dx} = 0.57 \frac{F}{l} \tag{3}$$

Thus, the resonance displacement of the cantilever is described by the expression

$$y(x,t) = \frac{Q \Phi}{\mu \omega^2} \psi(x) \sin\omega t \tag{4}$$

where $\mu = m/l$, and m is the mass of resonator.

From (1)-(4) we find the expression for the amplitude of resonator tip vibrations y_ω

$$y_\omega = 3.1 \frac{Q \varepsilon_0 u_\omega u_0}{h \rho \omega^2 d^2} \tag{5}$$

The oscillation of the resonator modulates the phase of interferometer output signal. The coefficient of conversion of applied voltage into the interferometer phase modulation is

$$S = \frac{\varphi_\omega}{u_0} = \frac{4\pi}{\lambda} \frac{y_\omega}{u_0} \tag{6}$$

where λ is the wavelength.

Substituting the parameters of electrometer into (5) and (6) we find the theoretical prediction of the conversion coefficient $S_t = 0.522$ rad/V, which appeared to be in a good correspondence with the coefficient $S_e = 0.519$ rad/V observed experimentally.

The fundamental reason limiting the sensitivity of the electrometer is thermal vibrations of the resonator.

The power spectral density for thermally generated vibrations can be derived from the fluctuation-dissipation theorem[7]. As a result we find the expression for the interferometer phase fluctuations generated thermally:

$$<\Delta\varphi^2>^{1/2} = \begin{cases} \dfrac{8\pi}{\lambda}\left(\dfrac{kT}{\eta}\right)^{1/2} & ;\ 2\pi\Delta f > \omega/Q \qquad (7.1) \\ \dfrac{16\pi}{\lambda}\left(\dfrac{kT}{\eta}\right)^{1/2}\left(\dfrac{kT}{m}\dfrac{\omega}{Q}\right)^{1/2} & ;\ 2\pi\Delta f < \omega/Q \qquad (7.2) \end{cases}$$

where $\eta = m\,\omega^2$ is the bend stiffness of the resonator beam, Δf is the frequency band of lock-in amplifier. In the first case described by (7.1) the electrometer response time depends upon the width of the resonance curve, while in the second case the response time depends only upon the parameters of the lock-in sircuit. As can be seen from formulas (7) the utilization of lock-in amplifier with bandwidth lower than the natural resonator band allows to improve sufficiently the electrometer threshold sensitivity.

With the aid of (5),(6) and (7.2) it is possible to find the electrometer threshold sensitivity limited by the thermal vibrations of the sensitive element. The electrometer operating at the resonant frequency ω with a detection bandwidth $2\pi\Delta f < \dfrac{\omega}{Q}$ exhibits a minimum detectable voltage described by the following equation

$$\frac{u_{\min}}{\sqrt{\Delta f}} = 1.3\,\frac{\rho\,h\,d^2}{\varepsilon_0\,u_\omega}\left(\frac{kT}{m}\frac{\omega}{Q}\right)^{1/2} \qquad (8)$$

Substituting the parameters of electrometer in formula (8) we find the minimal detectable voltage of $2\cdot 10^{-4}\mathrm{V}/(\mathrm{Hz})^{1/2}$. This thermal fluctuation limit coincides with the experimental result.

The alternating voltage may be increased up to flashover voltage of 3700 V in which case the sensitivity of $3\,\mu\mathrm{V}$ is predicted. The further improvement of the sensitivity may be achieved by diminishing of the resonance frequency and the distance between the capacitor plates. An order of magnitude estimations indicate that the charge close to the electron charge may be measured in this case.

5. Discussion

The aim of the research work presented in this paper was to demonstrate the operation of a new type of electrometer and to investigate its main characteristics. The simplest scheme of electrometer has been realized in which the regulation of the mean separation between the interferometer mirrors is accomplished by magnetic field deviating the resonator tip. The piezoceramic scheme of interferometer regulation was also investigated (Fig.3).

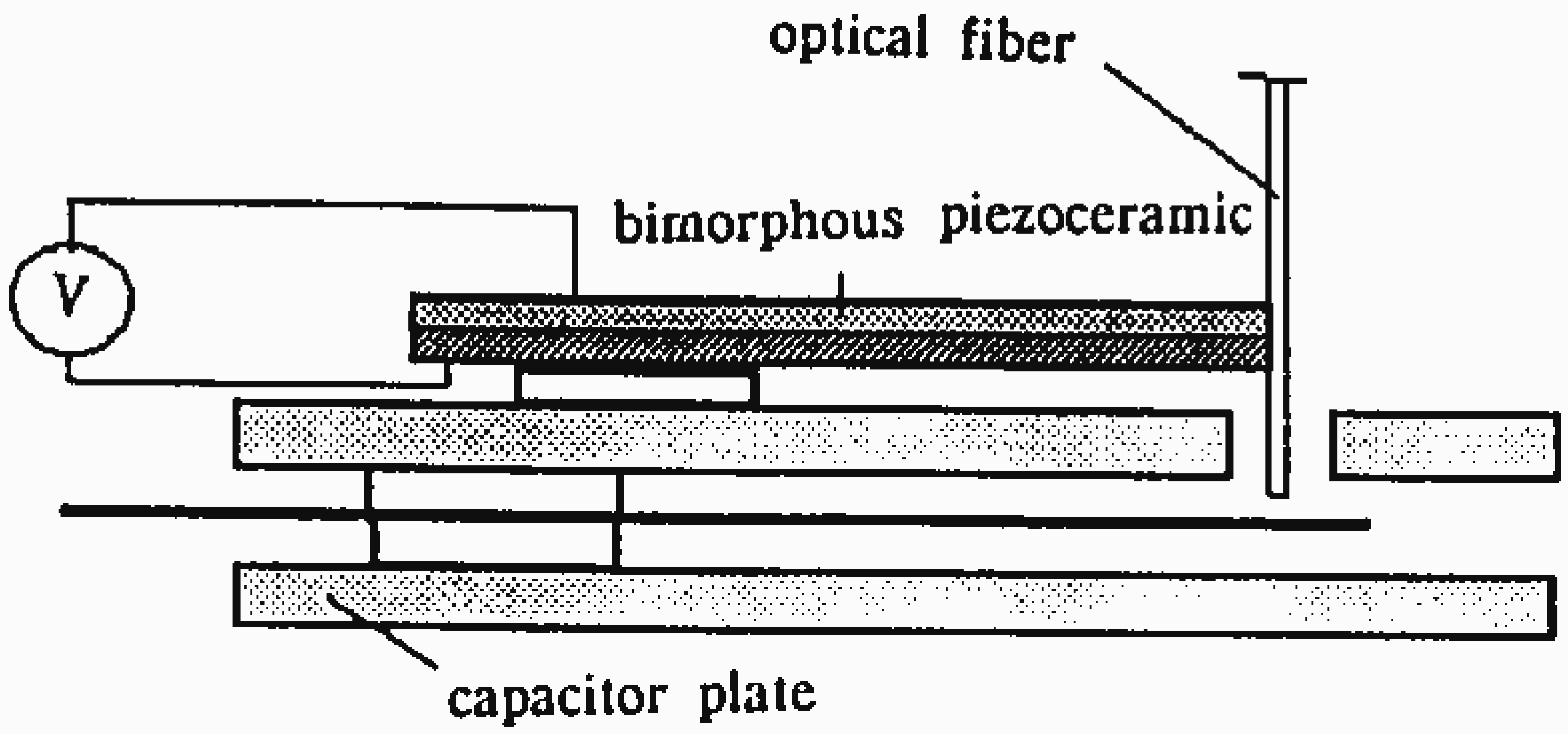

Fig.3. The piezoceramic scheme of interferometer regulation

The voltage of about 30 V changed the interferometer separation by 1 micrometer.

The electrometer configuration in which the sensitive resonator is placed between the capacitor plates makes it insensitive to the external electric fields. To the contrary, when the resonator was shifted out from the plates the sensitivity to the external electric fields was high and it is interesting for the further investigations to explore this effect in the context of creation of a new fiber optic sensor of electric field[8,9].

Limitations to the electrometer sensitivity may originate from several sources including as laser and photodetector noises, as an acoustic interference. Meanwhile, the thermal fluctuation limit is of a fundamental character since it can not be eliminated by any technical improvements. So, if the thermal noise is dominating, there is no sense to improve the interferometer performance or to isolate the resonator acoustically. In the case considered in this paper the value of the thermal fluctuation limit coincides with the noise observed in the experiment. It is not a simple technical problem to prove, that it was exactly the thermal noise that was prevailing in the experiment. Meanwhile, the foregoing analysis shows that the thermal vibrations of resonator is comparable with all other noise sources and inevitably will limit the sensitivity of further-generation electrometers of the considered type.

6. References

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